Discovering Math: Concepts in Geometry
Teacher's Guide

Grade Level: 10–12	**Curriculum Focus:** Mathematics	**Lesson Duration:** Three class periods

Program Description

Concepts in Geometry — Take a look at modern and ancient architecture to learn more about geometry and the properties of triangles, quadrilaterals, circles, and polygons. Includes one feature and four shorter segments.

Onscreen Questions

- How are triangles used in the design and construction of bridges?
- What are some examples of circular architectural structures?
- How did the Greeks use their knowledge of geometry to advance their culture?
- What contributions did Pythagoras make to geometry in addition to the Pythagorean Theorem?

Lesson Plan

Student Objectives

- Discover Greek achievements in mathematics.
- Understand deductive reasoning.
- Demonstrate mathematical proofs of selected theorems.
- Understand Euclid's proof of the Pythagorean Theorem.

Materials

- *Discovering Math: Concepts in Geometry* video
- Computer with Internet access
- Print resources geometry and ancient Greece

Procedures

1. Have students research different proofs of the Pythagorean Theorem and create a poster demonstrating one such proof using print and Web resources. The following Web sites are a good starting point:

   - Pythagorean Theorem and its many proofs

     http://www.cut-the-knot.org/pythagoras/index.shtml

   - Proofs of the Pythagorean Theorem
     http://jwilson.coe.uga.edu/EMT668/EMT668.Student.Folders/HeadAngela/essay1/Pythagorean.html

   - Pythagorean Theorem (at MathWorld)
     http://mathworld.wolfram.com/PythagoreanTheorem.html

   Have the students create a bulletin board from their posters.

2. Have students research a Greek mathematician using print and Web resources. The following Web sites are a good starting point:

   - Index of Greek Mathematicians

     http://www-groups.dcs.st-and.ac.uk/~history/Indexes/Greek_index.html

   - Greek Mathematics and its Modern Heirs
     http://www.ibiblio.org/expo/vatican.exhibit/exhibit/d-mathematics/Greek_math.html

   - Math Lair – Greek Mathematicians

     http://www.stormloader.com/ajy/greekdates.html

   - Ancient Greek Mathematics

     http://www.math.tamu.edu/~dallen/masters/Greek/readings4.htm

3. When students have completed their research, ask them to summarize their findings in a one-page report.

4. Have students choose a partner. Ask them to share their reports with their partners and answer any questions. Then have each student summarize his or her partner's report for the class, including at least three interesting facts.

5. Discuss indirect measurement with the students. On a sunny day, give students yardsticks. Have them measure their heights and shadows and record the results. Then, ask them to measure the shadows of several tall objects, such as trees or flagpoles, and record the results. Return to the classroom and demonstrate how to set up a proportion to find the height of a tall object from the length of its shadow. Then have the students use the shadow lengths of the tall objects they found outside to find the heights of those objects.

6. Have students keep a notebook to record the postulates, theorems, and proofs of selected theorems shown in the video. The following are postulates of Thales:

> A circle is bisected by any diameter.
>
> The base angles of an isosceles triangle are equal.
>
>> Have students help demonstrate the proof of this theorem as seen on the video.
>
> The angles between two intersecting straight lines are equal.
>
> Two triangles are congruent if they have two angles and the included side equal.
>
> An angle in a semicircle is a right angle.

Note: The following Web site is a good source on the life and accomplishments of Thales:

http://www.math.tamu.edu/~dallen/masters/Greek/thales.pdf

7. Ask students to demonstrate a proof of the theorem stating that the sum of the interior angles of a triangle is equal to 180°.

8. Ask students to demonstrate a proof of Euclid's Proposition 41: If a parallelogram has the same base with a triangle and is in the same parallels, then the parallelogram is double the triangle.

9. Ask students to demonstrate a proof of the Pythagorean Theorem.

Note: The following Web site shows Euclid's proof of the Pythagorean Theorem:

http://aleph0.clarku.edu/~djoyce/java/elements/bookI/propI47.html

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students were highly engaged in class discussions; produced complete reports, including all of the requested information; clearly demonstrated the ability to measure indirectly, and showed a complete understanding of using deductive reasoning in mathematical proofs.

- **2 points:** Students participated in class discussions; produced an adequate report, including most of the requested information; satisfactorily demonstrated the ability to measure indirectly, and showed a satisfactory understanding of using deductive reasoning in mathematical proofs.

- **1 point:** Students participated minimally in class discussions; created an incomplete report with little or none of the requested information; were not able to measure indirectly, or adequately use deductive reasoning in mathematical proofs.

Vocabulary

axiom
Definition: A statement accepted as true without proof
Context: The Greeks based the study of geometry on definitions and axioms.

deductive reasoning
Definition: The process of reasoning logically from given statements to a conclusion
Context: The Pythagorean Theorem was proved using deductive reasoning.

indirect measurement

Definition: A method of measuring distances by solving a proportion

Context: Thales used indirect measurement to determine the height of a pyramid.

logic

Definition: The formal principles of reasoning

Context: Each statement in a proof is logically justified by a definition, postulate, or an earlier proposition that has already been proven.

mathematical proof

Definition: A demonstration that a mathematical proposition is true based on axioms, definitions, and proven theorems

Context: Euclid's proof of the Pythagorean Theorem made use of the previous proven theorem known as Proposition 41.

theorem

Definition: A proposition that has been or is to be proved on the basis of certain assumptions

Context: In Book 1 of *Elements*, Euclid's proposition 41 is the theorem "if a parallelogram has the same base with a triangle and is in the same parallels, then the parallelogram is double the triangle."

Academic Standards

National Council of Teachers of Mathematics (NCTM)
The National Council of Teachers of Mathematics provides guidelines for teaching mathematics in grades K–12 to promote mathematical literacy. To view the standards, visit this Web site:
http://standards.nctm.org/document/chapter3/index.htm

This lesson plan addresses the following thematic standards:
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K–12 Education addresses 14 content areas. To view the standards and benchmarks, visit
http://www.mcrel.org/compendium/browse.asp .

This lesson plan addresses the following national standards:

- Mathematics: Understands and applies basic and advanced properties of the concepts of geometry; Use the Pythagorean theorem and its converse and properties of special right triangles to solve mathematical and real-world problems; Understands the basic concepts of right triangle trigonometry (e.g., basic trigonometric ratios such as sine, cosine, and

tangent); Uses trigonometric ratio methods to solve mathematical and real-world problems (e.g., determination of the angle of depression between two markers on a contour map with different elevations); Uses properties of and relationships among figures to solve mathematical and real-world problems (e.g., uses the property that the sum of the angles in a quadrilateral is equal to 360 degrees to square up the frame for a building; uses understanding of arc, chord, tangents, and properties of circles to determine the radius given a circular edge of a circle without the center)

- Science: Physical Science: Understands the structure and properties of matter; Understands the sources and properties of energy
- World History: Understands Greek achievements in mathematics
- Historical Understanding: Understands the impact and achievements of the Hellenistic period (e.g., major lasting achievements of Hellenistic art, mathematics, science, philosophy, and political thought; the impact of Hellenism on Indian art; how architecture in West Asia after the conquests of Alexander reflected Greek and Macedonian influence)

Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- http://school.discovery.com/teachingtools/teachingtools.html

DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How To Use the DVD

The DVD starting screen has the following options:

Play Video—This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index—Here the video is divided into sections indicated by video thumbnail icons; brief descriptions are noted for each one. Watching all parts in sequence is similar to watching the video from start to finish. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Curriculum Units—These are specially edited video segments pulled from different sections of the video (see below). These nonlinear segments align with key ideas in the unit of instruction. They include onscreen pre- and post-viewing questions, reproduced below in this Teacher's Guide. Total running times for these segments are noted. To play a particular segment, press Enter on the TV remote or click once on the Curriculum Unit title on a computer.

Standards Link—Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources—This screen gives the technical support number and Web site address.

Video Index

I. Triangles (5 min.)

Triangles are used in bridge design. Learn different types of triangles and how to find the area of a triangle. Discover how engineers use trigonometry to calculate measurements of structures.

II. Areas of Quadrilaterals (6 min.)

Different quadrilateral shapes can be found in skyscrapers. Learn how to calculate the surface area of part of a skyscraper. *Hint:* Find the areas of the quadrilaterals that make up this part's surface, and then add these areas.

III. Circles (5 min.)

Many circular architectural structures exist in the world. Learn how the diameter and circumference of a circle are related, and see how to find the measures of angles in a circle.

IV. Pentagons (6min.)

Learn fascinating mathematical facts about the Pentagon in Washington, D.C.

V. Math and the Greeks (26 min.)

The ancient Greeks had a great influence on philosophy and mathematics. Learn about some Greek philosophers and mathematicians and their contributions to the study of geometry.

Curriculum Units

1. Triangles

Pre-viewing question
Q: How are triangles used in everyday life?
A: Answers will vary.
Post-viewing question
Q: Why are triangles used in the design of some bridges?
A: Answers will vary.

2. Using Trigonometric Functions in Triangles

Pre-viewing question
Q: How are trigonometric functions used?
A: Answers will vary.
Post-viewing question
Q: How do you find the length x of the side opposite a 40° angle in a right triangle that has a hypotenuse of 54 meters?
A:

$$\sin 40° = \frac{x}{54}$$
$$x = 54 \cdot \sin 40°$$
$$x = 34.71 \text{ meters}$$

3. Areas of Quadrilaterals

Pre-viewing question
Q: What is a situation where you would you need to find the area of a quadrilateral?
A: Answers will vary.
Post-viewing question
Q: In the video, how was the surface area of the three-dimensional figure found?
A: Answers will vary.

4. Circles

Pre-viewing question
Q: How are circles used in everyday life?
A: Answers will vary.
Post-viewing question
Q: Quadrilateral *ABCD* is inscribed in circle *O* as shown.

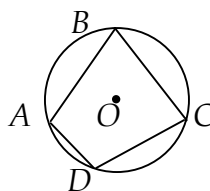Prove that $\angle A$ and $\angle C$ are supplementary.

A: Proof: $m\angle A = \frac{1}{2}m\overarc{BCD}$, $m\angle C = \frac{1}{2}m\overarc{BAD}$

$$m\angle A + m\angle C = \frac{\frac{1}{2}m\overarc{BCD}}{2} + \frac{\frac{1}{2}m\overarc{BAD}}{2}$$

$$\frac{1}{2}m\overarc{BCD} + \frac{1}{2}m\overarc{BAD} = 360°$$

$$m\angle A + m\angle C = \frac{1}{2} \cdot 360° = 180°$$

So, $\angle A$ and $\angle C$ are supplementary.

5. The Pentagon

Q: Can you name something in everyday life, other than the Pentagon, that has the shape of a regular polygon?
A: Answers will vary.
Post-viewing question
Q: What is a formula for finding the measure of an interior angle in a regular polygon? What is a formula for finding the area of a regular polygon of *n* sides?

A: The measure of an interior angle in a regular polygon of *n* sides is $\frac{(n-2)}{n}\cdot 180°$. The area of a

regular polygon of *n* sides is $A = \frac{1}{4}ns^2 \cot\left(\frac{180°}{n}\right)$, where *s* is the length of a side.

6. Greek Thought

Pre-viewing question
Q: How has Greek civilization affected your life?
A: Answers will vary.
Post-viewing question
Q: How were Babylonian and Egyptian methods in geometry different from those of the Greeks?
A: Answers will vary.

7. Thales

Pre-viewing question
Q: Name a well-known person in Greek history and the effect he had on world culture.
A: Answers will vary.
Post-viewing question
Q: How did Thales measure the height of pyramid?
A: Answers will vary.
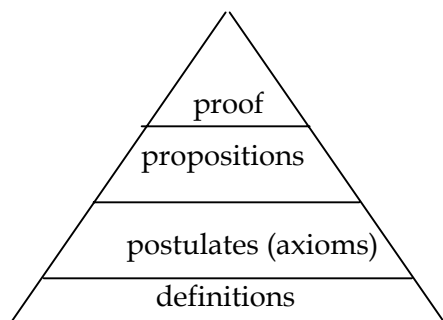
8. Euclid

Pre-viewing question
Q: Who was Euclid?
A: Answers will vary.
Post-viewing question
Q: Explain Euclid's methodology in geometry using a triangular graphic organizer.
A:

9. Euclid's Proposition 41

Pre-viewing question
Q: What is the formula for the area of a triangle and the formula for the area of a parallelogram?
A: The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the base of the triangle and h is the

height. The formula for the area of parallelogram is $A = bh$, where b is the base of the parallelogram and h is the height.

Post-viewing question
Q: What is Proposition 41 in Book One of Euclid's *Elements*?
A: If a parallelogram and triangle share the same base and are in the same parallels, then the area of the triangle is half the area of the parallelogram.

10. Pythagoras

Pre-viewing question
Q: Name a Greek philosopher other than Pythagoras.
A: Answers will vary.
Post-viewing question
Q: Describe the philosophy of Pythagoras.
A: Answers will vary.

11. Pythagorean Theorem

Pre-viewing question
Q: How do you use the Pythagorean theorem?
A: Answers will vary.
Post-viewing question
Q: Outline the proof of the Pythagorean theorem.
A: Answers will vary.